

## 12. Geometric figures

### ASTRONOMY

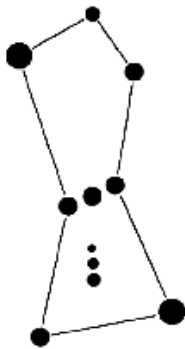
Astronomy is a natural science that deals with the study of celestial objects (such as stars, planets, comets, nebulae, star clusters and galaxies)

It is one of the oldest sciences. Astronomy became a modern science with the invention of the telescope in the 17th Century.

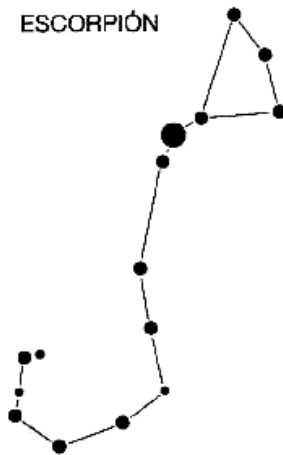
It was a fundamental subject to determine the seasons, the calendar, the time and the position on the ground and more important in the sea, that is, for the sailors.

Here you have some of the most important constellations you can see in the sky at a glance.

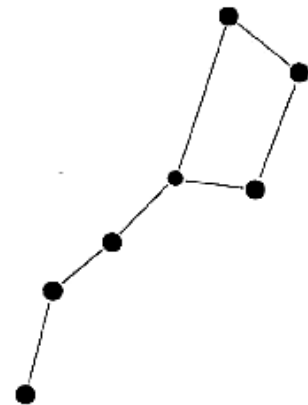
ORIÓN



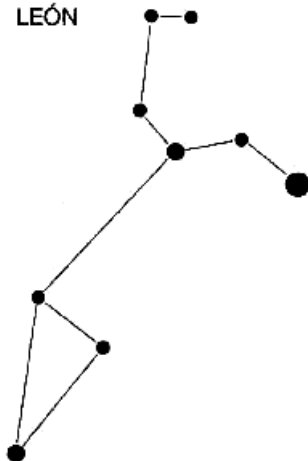
ESCORPIÓN



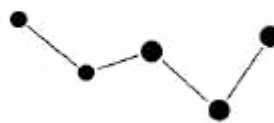
OSA MAYOR



LEÓN



CASIOPEA



OSA MENOR

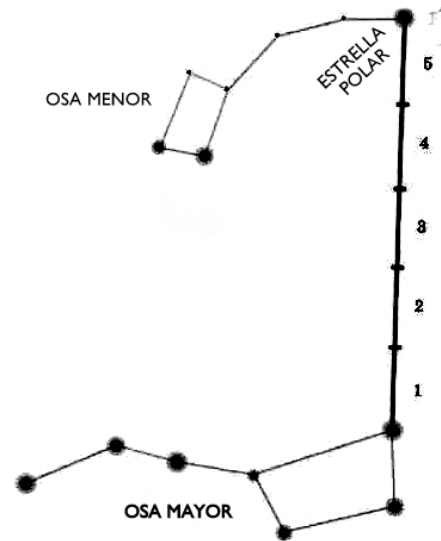


## HOW TO LOCATE THE NORTH STAR?

The Polar star points us to the geographical North so it is the most important star in the sense of orientation.

Here you have the way to locate the Polar Star in the sky. You have to find the Ursa Major (Big Dipper), this is very easy because it is a very big constellation and it is always present in the sky.

After that you have to extend imaginarily the ending part five times and you will find a star of medium brightness. This is the Polar Star. From here you can easily find the rest of stars that form the Little Dipper.



## 1. CONSTRUCTION OF TRIANGLES

A triangle is a polygon with three sides. We name the sides with lowercases and the angles with uppercases.

There is a unique triangle with:

1. Three sides $a$ , $b$ and $c$ . The longest side has to be smaller than the sum of the others.	
2. Two sides $a$ and $b$ and the angle formed by them.	
3. A side $a$ and the angles which are on it. The sum of these angles has to be less than $180^\circ$ .	

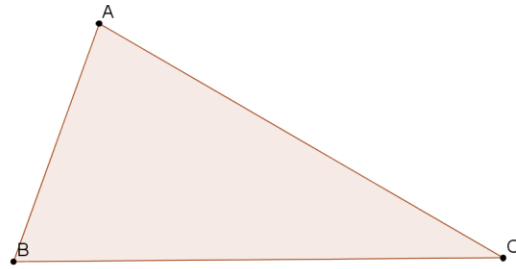
The triangle is unchangeable.

## 2. IMPORTANT SEGMENTS IN A TRIANGLE

### MEDIAN

The median is the segment that joins a vertex with the midpoint of the opposite side.

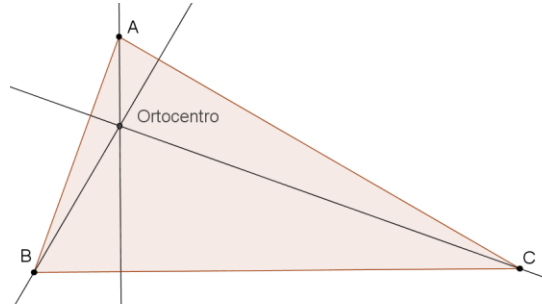
The three medians have a common point called barycentre. This is the centre of gravity of the triangle.



### HEIGHT

The height is the segment that joins a vertex with the opposite side in a perpendicular way.

The three heights have a common point called orthocentre.

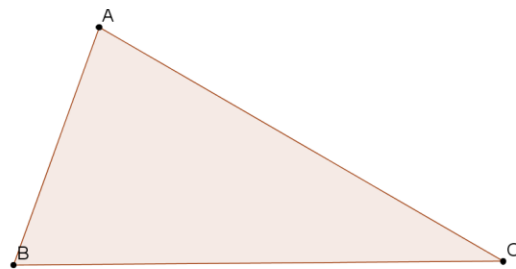


### PERPENDICULAR BISECTOR

The line that passes through the midpoint of a segment and it is perpendicular to the segment.

The three perpendicular bisectors of a triangle have a common point called circumcentre.

This is the centre of the circumcircle of the triangle.

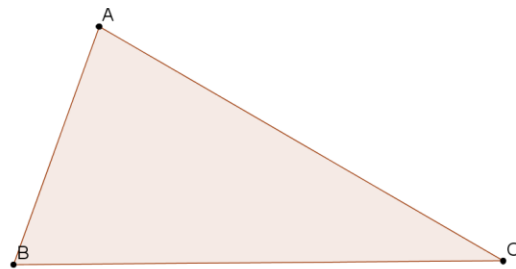


### ANGLE BISECTOR

The line that divides the angle into two equal parts.

The three angle bisectors have a common point called incentre.

This is the centre of the incircle of the triangle.



## 3. QUADRILATERALS

A quadrilateral is a polygon with four sides.

## TYPES OF QUADRILATERALS

### **Parallelograms**

A parallelogram is a quadrilateral which has parallel sides in pairs.

<b><u>Square</u></b> A square has 4 equal sides and 4 right angles.	
<b><u>Rectangle</u></b> A rectangle has equal opposite sides and 4 right angles.	
<b><u>Rhombus</u></b> A rhombus has four equal sides and equal opposite angles.	
<b><u>Rhomboid</u></b> A rhomboid has opposite sides that are parallel.	

### **Trapezium**

Trapezium is quadrilateral with only two parallel sides. They are classified as follows:

<b><u>Right trapezium</u></b> A right trapezium has two right angles.	
<b><u>Isosceles trapezium</u></b> An isosceles trapezium has two of its non-parallel sides equal in length.	

**Scalene trapezium**

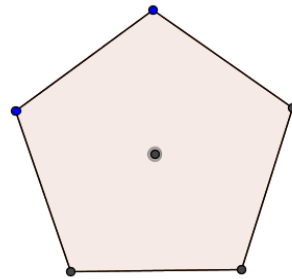
A scalene trapezium doesn't have equal sides or angles.

**Trapezoid**

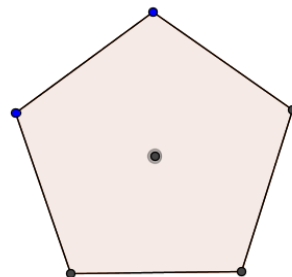
It is a quadrilateral with no parallel sides.

**4. REGULAR POLYGON*****CENTRE***

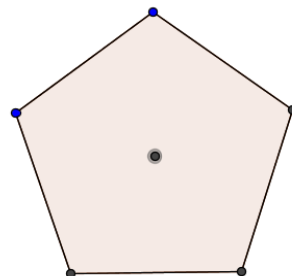
The centre of a regular polygon is the point that is the same distance from the vertices.

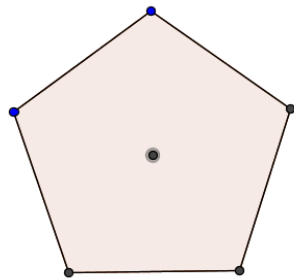
***Radius***

The radius of a regular polygon is the segment which joins the centre with whatever vertex.

***APOTHEM***

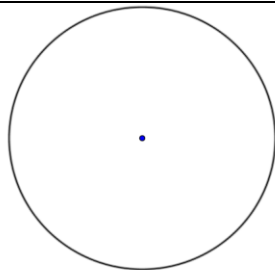
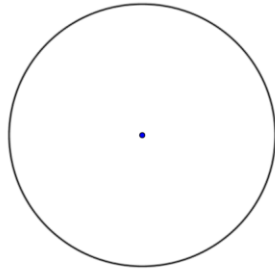
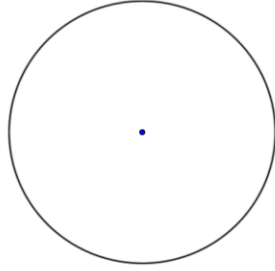
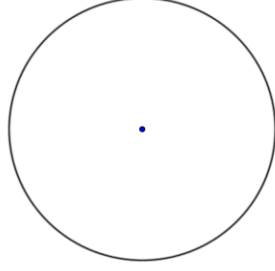
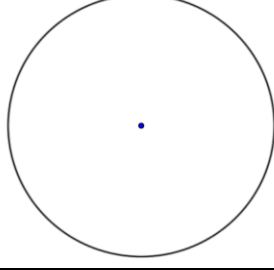
The apothem is the segment that joins the centre with a side in a perpendicular way.

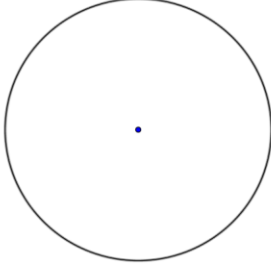


<p><b>The side, the radio and the apothem</b></p> <p>There is the following relation:</p> $R^2 = a^2 + \left(\frac{l}{2}\right)^2$	
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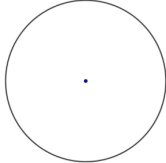
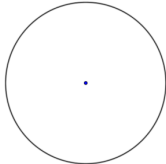
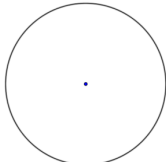
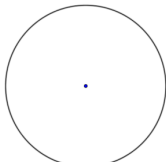
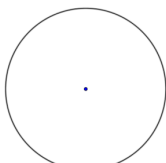
## 5. CIRCUMFERENCE

A circumference is a closed curved line whose points are all the same distance from a fixed point called the centre.

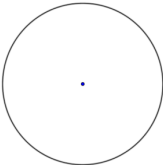
<p><b>Centre of a circumference</b></p> <p>The centre is the point which is equidistant from all points in the circumference.</p>	
<p><b>Radius of the circle</b></p> <p>The radius is the distance from the centre to the edge of the circle.</p>	
<p><b>Chord</b></p> <p>A chord is a line segment that joins two points in the circumference.</p>	
<p><b>Diameter</b></p> <p>The diameter is a chord that passes through the centre of the circumference.</p>	
<p><b>Arc</b></p> <p>The part of a circumference between two points.</p>	

<p><b>Semicircumference</b></p> <p>A semi-circumference is each of the two equal parts in which a diameter divides a circumference.</p>	
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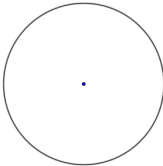
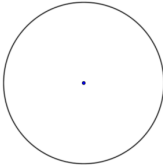
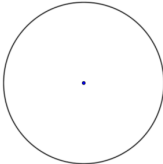
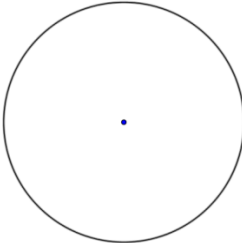
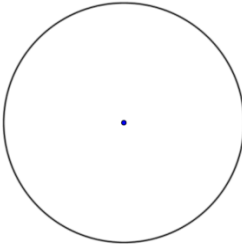
#### POSITION REGARDING A CIRCUMFERENCE

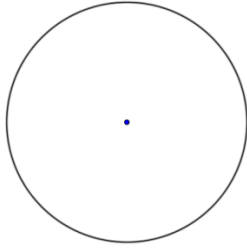
<p><b>Point outside the circumference</b></p> <p>If the distance from the point to the centre point of the circumference is greater than the length of the radius.</p>	
<p><b>Point on the circumference</b></p> <p>If the distance from the point to the centre of the circumference is exactly equal to the length of the radius of the circumference.</p>	
<p><b>Point within the circumference</b></p> <p>If the distance from the point to the centre of the circumference is less than the length of the radius.</p>	
<p><b>Exterior line to the circumference</b></p> <p>If they have no point in common and the distance from the centre to the line is greater than the length of the radius.</p>	
<p><b>Tangent line to the circumference</b></p> <p>If the line touches at a point. A line tangent to a circumference is tangent to the radius at this place.</p>	



<p><b><i>Secant line to the circumference</i></b></p> <p>If they have two common points.</p>	
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#### RELATIVE POSITIONS OF TWO CIRCUMFERENCES IN THE PLANE

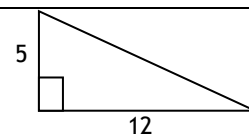
<p><b><u>Externals</u></b></p> <p>If they do not overlap and all the points are external.</p>	
<p><b><i>External tangent</i></b></p> <p>If they have just one common point and all other points are exterior to the other.</p>	
<p><b><u>Secants</u></b></p> <p>If they intersect at two different points.</p>	
<p><b><u>Internal tangent.</u></b></p> <p>If they have a common point and all other points of one of them are interior to the other, exclusively.</p>	
<p><b><u>Eccentric interior.</u></b></p> <p>If they have no common points and one is within the other.</p>	

<p><b><u>Concentrics</u></b></p> <p>If they have the same centre.</p>	
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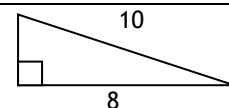
## 6. PYTHAGORAS'S THEOREM

The sides of a right triangle are called:  
Hypotenuse (h) and catheti or legs (a and b).  
The theorem establishes that:  $h^2 = a^2 + b^2$   
Three numbers that verify the below equality are called  
Pythagorean triple.

Calculate the unknown side



Calculate the unknown side



## 7. POLYHEDRONS

### DEFINITIONS

A polyhedron is the solid which is limited by polygons.

Face is each polygon.

Edge is each segment that is between two consecutive faces.

Vertex is each point that is common to many edges.

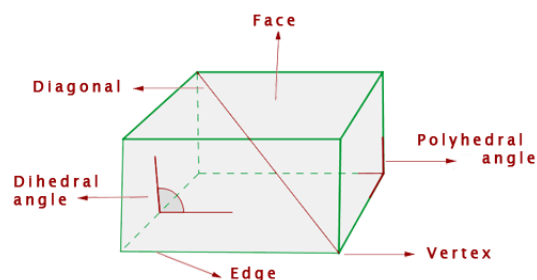
### ***Euler's formula***

It is verified that in all convex polyhedrons:

The number of faces plus the number of vertices equals the number of edges plus 2.

**No. of faces + No. of vertices = No. of edges + 2.**

$$F + V = E + 2$$



## TYPES OF POLYHEDRA

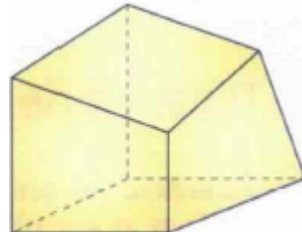
### Regular

The faces are equal regular polygons. The platonic solids are convex regular polyhedrons.



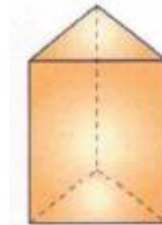
### Irregular

It has irregular polygons as faces.



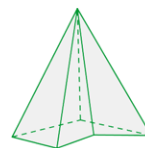
### Convex

The dihedral angles are less than  $180^\circ$ . The line segment joining any two points of the polyhedron is contained in the interior or in the face.



### Concave

It is the polyhedron that has a dihedral angle higher than  $180^\circ$ .

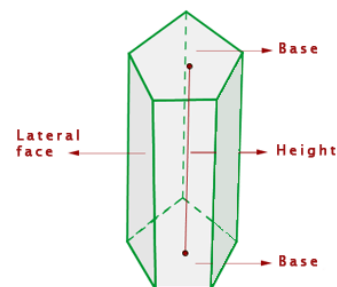


## PRISMS AND CYLINDERS

### PRISM

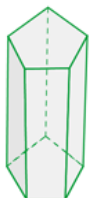
A prism is a polyhedron with two parallel and equal faces named bases. The other faces are the parallelograms that join these faces. They are the lateral faces.

The height is the distance between the bases.



### TYPES OF PRISMS

#### Regular Prisms



The bases are regular polygons

#### Irregular Prisms



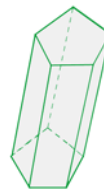
The bases are irregular polygons

#### Right Prisms



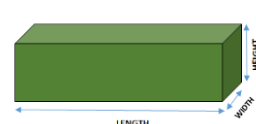
The lateral faces are rectangles or squares

#### Oblique Prisms



The lateral faces are rhomboids or rhombi.

#### Cuboid

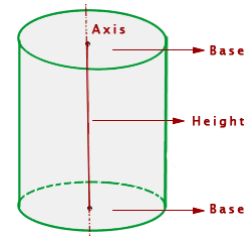


All the lateral faces are rectangles

## CYLINDERS

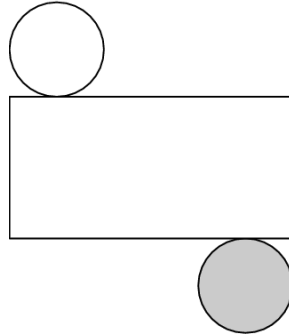
A curve solid is the solid which has some curve face.

The cylinder is the solid we get by rotating a rectangle around one of its faces.



## UNFOLDING OF A CYLINDER

The unfolding of a right cylinder is formed by two circles -the bases- and a rectangle -the lateral surface-.



## PYRAMIDS AND CONES

### Definition

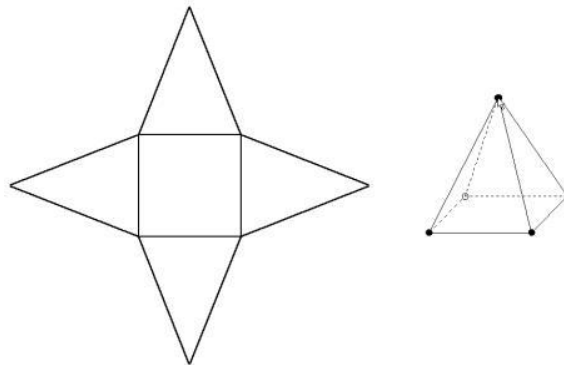
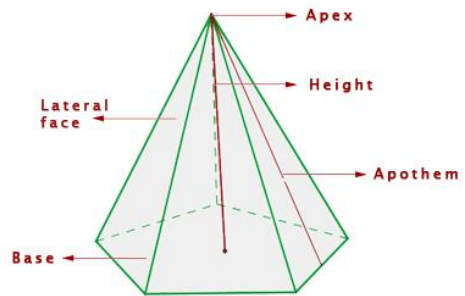
Pyramid is a polyhedron which faces are the following:

A polygon named base and the triangles formed joining the vertices with a fixed point named the vertex or apex of the pyramid.

The height is the distance between the vertex and the base.

### Unfolding of a Pyramid

The unfolding of a regular pyramid is formed by a regular polygon -the base- and as many isosceles and congruent triangles as lateral faces.



## Types of Pyramids

**Regular Pyramid**



The base of a regular pyramid is a regular polygon and its lateral faces are equal.

**Irregular Pyramid**



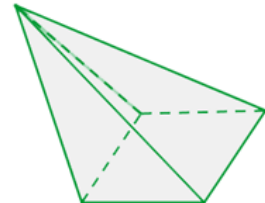
The base is an irregular polygon, and as a result, its faces are not equal.

**Right Pyramid**



it has isosceles triangles as its lateral faces and its apex lies directly above the midpoint of the base.

**Oblique Pyramid**



An oblique pyramid does not have all isosceles triangles as its faces.

## CONES

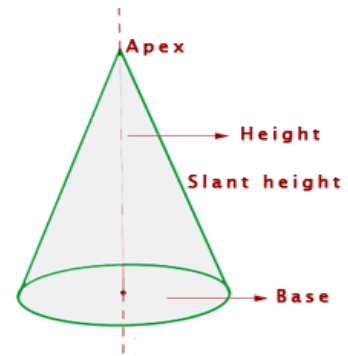
### Definitions

A right cone is a solid of revolution generated by a right triangle that rotates around one of its legs.

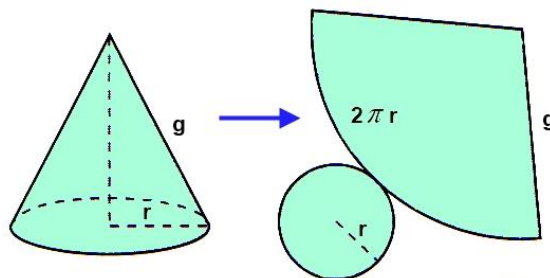
The height is the distance between the vertex and the base.

Generatrix or slant height is the segment from the vertex to a circle point.

In a right cone there is a Pythagorean relation:  
 $G^2 = R^2 + H^2$



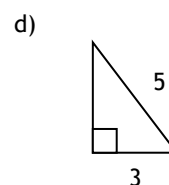
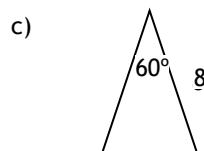
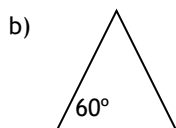
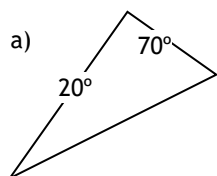
### Unfolding of a right cone



## EXERCISES AND PROBLEMS

### 1. Construction of triangles

1. Classify the following triangles by their sides and their angles. You have to calculate all of them to have a correct conclusion. **The pictures are not exactly, they are only a way to show the information.**



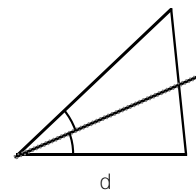
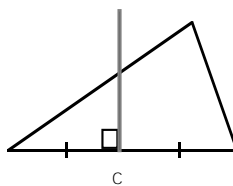
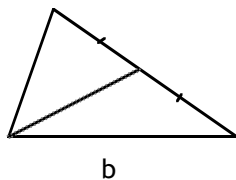
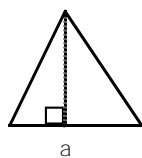
2. Is it possible to draw a triangle with the following sides: 6 cm, 12 cm y 13 cm? Explain your answer.

<p>3. Make a triangle whose sides are 3, 5 y 7 cm. What kind of triangle is it?</p>	
<p>4. Do the same as above with the triangle: 2, 5 y 8. What type of triangle is it?</p>	
<p>5. Draw a triangle whose base is 4 and the angles over it are each 60 degrees. What triangle is it?</p>	
<p>6. The base of a triangle is 5 cm and it has angles of 120° and 10° over it. Draw a picture and classify the triangle.</p>	


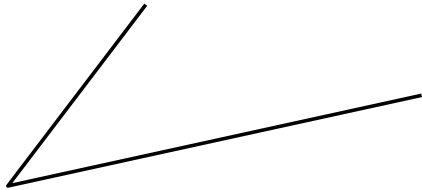
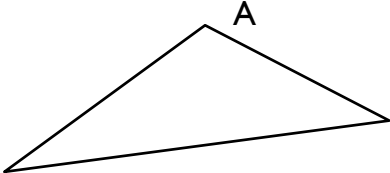
<p>7. Draw a triangle whose sides are 4 cm and 4 cm and the angle between them is 60 degrees. What type of triangle is it?</p>	
<p>8. Draw a triangle with the following data: <math>A = 110^\circ</math>, <math>b = 7</math> cm y <math>c = 5</math> cm</p>	
<p>9. Draw a triangle with the following data: <math>A = 100^\circ</math>, <math>B = 30^\circ</math> y <math>c = 5</math> cm.</p>	
<p>10. Draw using a ruler a right-angled triangle whose sides are: 7cm, 8cm y 10cm respectively.</p>	

## 2. Important segment in a triangle

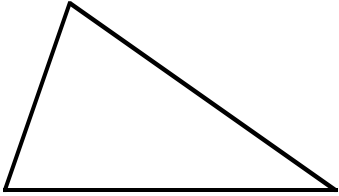
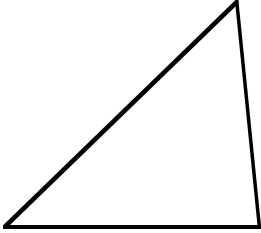
11. What is the segment which is drawn in each triangle:



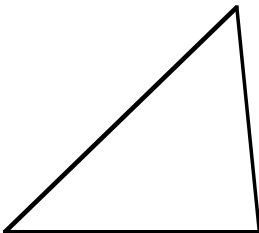
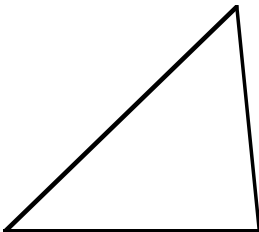
**12.** Draw what is asked in each part:

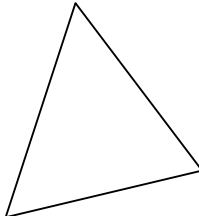
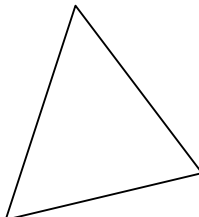
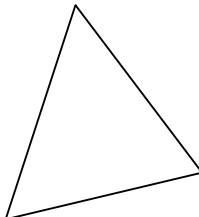
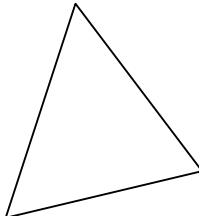
a. The perpendicular bisector of this segment	
b. The angle bisector of this angle	
c. The height of A:	

<b>13.</b> Using rulers and compasses draw two heights.	
<b>14.</b> Using rulers and compasses draw inside the triangle the three perpendicular lines bisectors.	



<p><b>15.</b> Using rulers and compasses draw the three medians.</p>	
<p><b>16.</b> Using rulers and compasses draw two angle bisectors.</p>	

<p><b>17.</b> Find the orthocentre. It is enough to draw two heights.</p>	
<p><b>18.</b> Find the incircle. It is enough to draw two bisector angles.</p>	
<p><b>19.</b> Find the circumcircle.</p>	
<p><b>20.</b> Find the barycentre</p>	

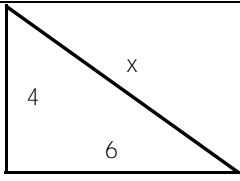
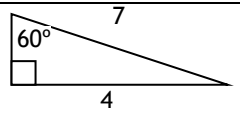
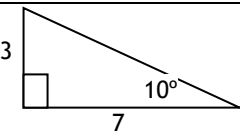
21. Draw with a ruler the incircle of a right-angled triangle which sides are: 6cm, 8cm and 10cm.	
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### 3. Quadrilaterals

### 4. Regular polygons

### 5. Circumference

### 6. Pythagoras Theorem

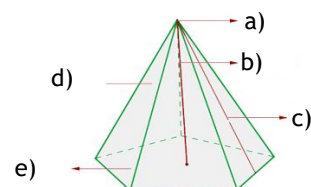
22. Calculate the unknown side in this triangle:	
23. Solve the next triangle. That is, find all its missing angles and sides. The small square means right angle, that is, 90°.	
24. Solve the next triangle. That is, find all its missing angles and sides.	

25. A triangle has sides of 10, 12 and 8. Is it a right triangle?
26. What is the height of an equilateral triangle with a side of 10 cm?
27. What is the diagonal of a rectangle whose sides are 7 cm and 5cm?
28. What value do you need to form a right-angled triangle with these sides: 5 cm, 12 cm?
29. What is the height of an isosceles triangle with a base of 4 cm and the equal sides are each 6 cm?

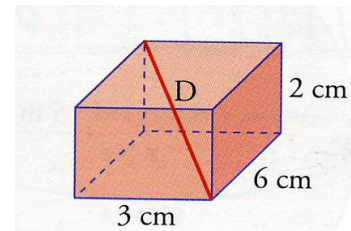
### 7. Polyhedrons

30. Complete the gaps in the figure with a suitable word.

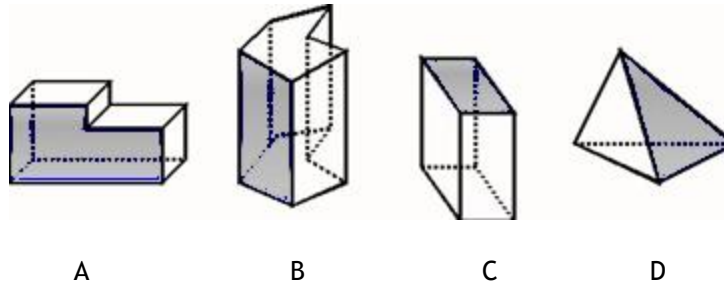
- a)
- b)
- c)
- d)
- e)



31. Work out the diagonal of the cuboid, the volume and the surface knowing that the length is 3 cm, the width is 6 cm and the height is 2 cm.



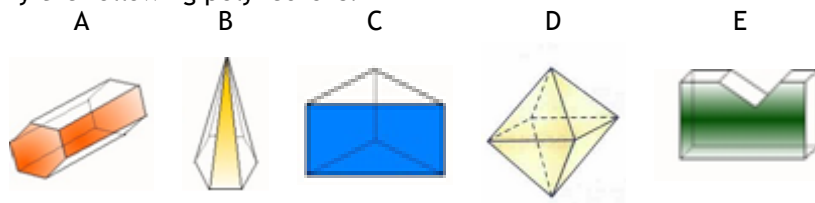
32. Classify the following polyhedrons:



- a) You have to write: concave or convex; regular or irregular.  
b) What kind of polyhedron are B, C and D respectively?  
c) Complete the table and verify Euler's formula?

	N° of edges	N° of faces	N° of vertices	Euler's formula
A				
B				
C				
D				

33. Classify the following polyhedrons:



- a. What are concave polyhedrons?  
b. What kind of polyhedrons are according to the number of faces? (only A to D)  
c. What kind of polyhedrons are according to their shapes? (only A to D)  
d. Check the Euler's formula for A, B and C.